C 40608 **(Pages : 2)** **Name.....................................**

Reg. No.................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS–UG)

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

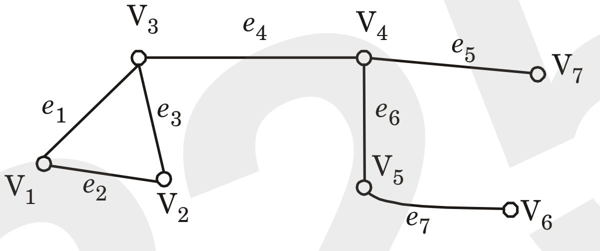
(2019 Admission onwards)

Time : Two Hours Maximum Marks : 60

Section A (Short Answer Questions)

Each question carries 2 marks.
A maximum of 20 marks can be earned from this section.

1. Find the number of edges of K_7.
2. Draw the graph $K_{3,3} - \{V\}$ where V is a Vertex in $K_{3,3}$.
3. Define K-regular graph. Give an example.
4. Define union of two graphs G_1 and G_2.
5. Let G be a simple graph with n vertices and $\overline{G}$ be its complement. Prove that, for each vertex V in G, $d_G(V) + d_{\overline{G}}(V) = n - 1$.
6. Define the adjacency matrix of a graph G with n vertices.
7. A connected graph G has 17 edges, what is the maximum possible number of vertices in G ?
8. Let



Find all bridges in G.

Turn over

9. When can you say that the complete graph $K_n, n \geq 3$ is Euler ? Justify.
10. Define critical planar graphs. Which complete graph K_n are critical planar ?
11. State Cayley's theorem on spanning trees.
12. When can you say that a graph G is maximal non-Hamiltonian.

(Ceiling marks = 20 marks)

Section B (Paragraph/Problem Type Questions)

Each question carries 5 marks.
A maximum of 30 marks can be earned from this section.

13. Prove that the complete bipartite graph $K_{3,3}$ is non-planar.
14. Let G be a simple planar graph with less than 12 vertices. Prove that G has a vertex V with $d(v) \leq 4$.
15. Prove that, in any graph G there is an even number of odd vertices.
16. Prove that for any connected graph G, $\text{rad G} \leq \text{diam G} \leq 2. \text{rad G}$.
17. Let u and v be distinct vertices of a tree 7. then prove that theme is precisely one path from u to v.
18. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.
19. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.

(Ceiling marks = 30 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 marks.

20. Explain the Konigsberg bridge problem. Give the graph theory model for this problem. Also state the respective theorem to solve this problem.
21. Let G be a graph with n vertices. Then prove that the following statements are equivalent :
    (i) G is a tree.
    (ii) G is acyclic graph with $n - 1$ edges.
    (iii) G is a connected graph with $n - 1$ edges.

(1 × 10 = 10 marks)